Standardized Test Questions

36. True or False If $f(x) = x^2 + x$, then $f'(x)$ exists for every real number x. Justify your answer.

37. True or False If the left-hand derivative and the right-hand derivative of f exist at $x = a$, then $f'(a)$ exists. Justify your answer.

38. Multiple Choice Let $f(x) = 4 - 3x$. Which of the following is equal to $f'(-1)$?

(A) -7 (B) 7 (C) -3 (D) 3 (E) does not exist

39. Multiple Choice Let $f(x) = 1 - 3x^2$. Which of the following is equal to $f'(1)$?

(A) -6 (B) -5 (C) 5 (D) 6 (E) does not exist

In Exercises 40 and 41, let

$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2x - 1, & x \geq 0. \end{cases}$$

40. Multiple Choice Which of the following is equal to the left-hand derivative of f at $x = 0$?

(A) -2 (B) 0 (C) 2 (D) ∞ (E) $-\infty$

41. Multiple Choice Which of the following is equal to the right-hand derivative of f at $x = 0$?

(A) -2 (B) 0 (C) 2 (D) ∞ (E) $-\infty$

Explorations

42. Let $f(x) = \begin{cases} x^2, & x \leq 1 \\ 2x, & x > 1. \end{cases}$

(a) Find $f'(x)$ for $x < 1$. (b) Find $f'(x)$ for $x > 1$.

(c) Find $\lim_{x \to 1^-} f'(x)$. (d) Find $\lim_{x \to 1^+} f'(x)$.

(e) Does $\lim_{x \to 1} f'(x)$ exist? Explain.

(f) Use the definition to find the left-hand derivative of f at $x = 1$ if it exists.

(g) Use the definition to find the right-hand derivative of f at $x = 1$ if it exists.

(h) Does $f'(1)$ exist? Explain.

43. Group Activity Using graphing calculators, have each person in your group do the following:

(a) pick two numbers a and b between 1 and 10;

(b) graph the function $y = (x - a)(x + b)$;

(c) graph the *derivative* of your function (it will be a line with slope 2);

(d) find the y-intercept of your derivative graph.

(e) Compare your answers and determine a simple way to predict the y-intercept, given the values of a and b. Test your result.

Extending the Ideas

44. Find the unique value of k that makes the function

$$f(x) = \begin{cases} x^3, & x \leq 1 \\ 3x + k, & x > 1 \end{cases}$$

differentiable at $x = 1$.

45. *Generating the Birthday Probabilities* Example 5 of this section concerns the probability that, in a group of n people, at least two people will share a common birthday. You can generate these probabilities on your calculator for values of n from 1 to 365.

Step 1: Set the values of N and P to zero:

```
0→N:0→P
                    0
```

Step 2: Type in this single, multi-step command:

```
N+1→N:1-(1-P)(366
-N)/365→P:{N,P}
```

Now each time you press the ENTER key, the command will print a new value of N (the number of people in the room) alongside P (the probability that at least two of them share a common birthday):

```
               {1 0}
   {2 .002739726}
   {3 .0082041659}
   {4 .0163559125}
   {5 .0271355737}
   {6 .0404624836}
   {7 .0562357031}
```

If you have some experience with probability, try to answer the following questions without looking at the table:

(a) If there are three people in the room, what is the probability that they all have *different* birthdays? (Assume that there are 365 possible birthdays, all of them equally likely.)

(b) If there are three people in the room, what is the probability that at least two of them share a common birthday?

(c) Explain how you can use the answer in part (b) to find the probability of a shared birthday when there are *four* people in the room. (This is how the calculator statement in Step 2 generates the probabilities.)

(d) Is it reasonable to assume that all calendar dates are equally likely birthdays? Explain your answer.

3.2 Differentiability

What you will learn about . . .

- How $f'(a)$ Might Fail to Exist
- Differentiability Implies Local Linearity
- Numerical Derivatives on a Calculator
- Differentiability Implies Continuity
- Intermediate Value Theorem for Derivatives

and why . . .

Graphs of differentiable functions can be approximated by their tangent lines at points where the derivative exists.

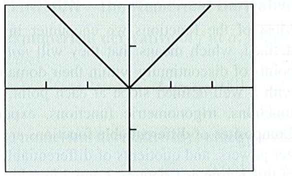
How $f'(a)$ Might Fail to Exist

A function will not have a derivative at a point $P(a, f(a))$ where the slopes of the secant lines,

$$\frac{f(x) - f(a)}{x - a},$$

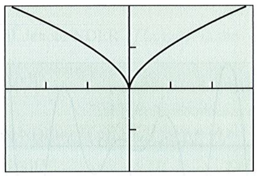
fail to approach a limit as x approaches a. Figures 3.11–3.14 illustrate four different instances where this occurs. For example, a function whose graph is otherwise smooth will fail to have a derivative at a point where the graph has

1. a *corner,* where the one-sided derivatives differ; Example: $f(x) = |x|$



[-3, 3] by [-2, 2]

Figure 3.11 There is a "corner" at $x = 0$.

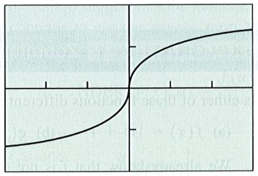


[-3, 3] by [-2, 2]

Figure 3.12 There is a "cusp" at $x = 0$.

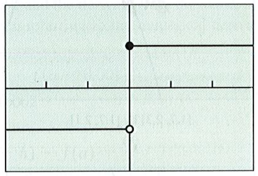
2. a *cusp,* where the slopes of the secant lines approach ∞ from one side and $-\infty$ from the other (an extreme case of a corner); Example: $f(x) = x^{2/3}$

3. a *vertical tangent,* where the slopes of the secant lines approach either ∞ or $-\infty$ from both sides (in this example, ∞); Example: $f(x) = \sqrt[3]{x}$



[-3, 3] by [-2, 2]

Figure 3.13 There is a vertical tangent line at $x = 0$.



[-3, 3] by [-2, 2]

Figure 3.14 There is a discontinuity at $x = 0$.

4. a *discontinuity* (which will cause one or both of the one-sided derivatives to be nonexistent). Example: The *Unit Step Function*

$$U(x) = \begin{cases} -1, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

In this example, the left-hand derivative fails to exist:

$$\lim_{h \to 0^-} \frac{U(0 + h) - U(0)}{h} = \lim_{h \to 0^-} \frac{(-1) - (1)}{h} = \lim_{h \to 0^-} \frac{-2}{h} = \infty.$$

How rough can the graph of a continuous function be?

The graph of the absolute value function fails to be differentiable at a single point. If you graph $y = \sin^{-1}(\sin(x))$ on your calculator, you will see a continuous function with an *infinite* number of points of nondifferentiability. But can a continuous function fail to be differentiable at *every* point?

The answer, surprisingly enough, is yes, as Karl Weierstrass showed in 1872. One of his formulas (there are many like it) was

$$f(x) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n \cos(9^n \pi x),$$

a formula that expresses f as an infinite (but converging) sum of cosines with increasingly higher frequencies. By adding wiggles to wiggles infinitely many times, so to speak, the formula produces a function whose graph is too bumpy in the limit to have a tangent anywhere!

Later in this section we will prove a theorem that states that a function *must* be continuous at a to be differentiable at a. This theorem would provide a quick and easy verification that U is not differentiable at $x = 0$.

EXAMPLE 1 Finding Where a Function Is Not Differentiable

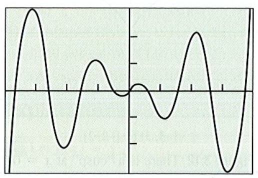
Find all points in the domain of $f(x) = |x - 2| + 3$ where f is not differentiable.

SOLUTION

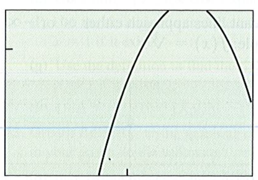
Think graphically! The graph of this function is the same as that of $y = |x|$, translated 2 units to the right and 3 units up. This puts the corner at the point $(2, 3)$, so this function is not differentiable at $x = 2$.

At every other point, the graph is (locally) a straight line and f has derivative $+1$ or -1 (again, just like $y = |x|$). ***Now Try Exercise 1.***

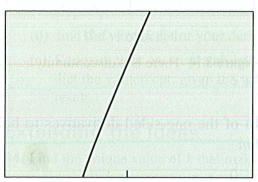
Most of the functions we encounter in calculus are differentiable wherever they are defined, which means that they will *not* have corners, cusps, vertical tangent lines, or points of discontinuity within their domains. Their graphs will be unbroken and smooth, with a well-defined slope at each point. Polynomials are differentiable, as are rational functions, trigonometric functions, exponential functions, and logarithmic functions. Composites of differentiable functions are differentiable, and so are sums, products, integer powers, and quotients of differentiable functions, where defined. We will see why all of this is true as Chapters 3 and 4 unfold.



[−4, 4] by [−3, 3]
(a)



[1.7, 2.3] by [1.7, 2.1]
(b)



[1.93, 2.07] by [1.85, 1.95]
(c)

Figure 3.15 Three different views of the differentiable function $f(x) = x\cos(3x)$. We have zoomed in here at the point $(2, 1.9)$.

Differentiability Implies Local Linearity

A good way to think of differentiable functions is that they are **locally linear;** that is, a function that is differentiable at a closely resembles its own tangent line very close to a. In the jargon of graphing calculators, differentiable curves will "straighten out" when we zoom in on them at a point of differentiability. (See Figure 3.15.)

EXPLORATION 1 Zooming in to "See" Differentiability

Is either of these functions differentiable at $x = 0$?

(a) $f(x) = |x| + 1$ (b) $g(x) = \sqrt{x^2 + 0.0001} + 0.99$

1. We already know that f is not differentiable at $x = 0$; its graph has a corner there. Graph f and zoom in at the point $(0, 1)$ several times. Does the corner show signs of straightening out?
2. Now do the same thing with g. Does the graph of g show signs of straightening out? We will learn a quick way to differentiate g in Section 3.6, but for now suffice it to say that it *is* differentiable at $x = 0$, and in fact has a horizontal tangent there.
3. How many zooms does it take before the graph of g looks exactly like a horizontal line?
4. Now graph f and g *together* in a standard square viewing window. They appear to be identical until you start zooming in. The differentiable function eventually straightens out, while the nondifferentiable function remains impressively unchanged.

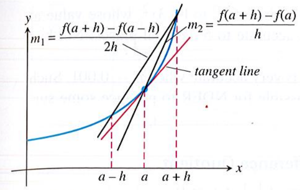


Figure 3.16 The symmetric difference quotient (slope m_1) usually gives a better approximation of the derivative for a given value of h than does the regular difference quotient (slope m_2), which is why the symmetric difference quotient is used in the numerical derivative.

Numerical Derivatives on a Calculator

For small values of h, the difference quotient

$$\frac{f(a+h) - f(a)}{h}$$

is often a good numerical approximation of $f'(a)$. However, as suggested by Figure 3.16, the same value of h will usually yield a *better* approximation of $f'(a)$ if we use the **symmetric difference quotient**

$$\frac{f(a+h) - f(a-h)}{2h}$$

to compute the slope between two nearby points on opposite sides of a. In fact, this approximation (which calculators can easily compute) is close enough to be used as a substitute for the derivative at a point in most applications.

DEFINITION The Numerical Derivative

The **numerical derivative of f at a,** which we will denote NDER $(f(x), a)$, is the *number*

$$\frac{f(a + 0.001) - f(a - 0.001)}{0.002}.$$

The **numerical derivative of f,** which we will denote NDER $(f(x), x)$, is the *function*

$$\frac{f(x + 0.001) - f(x - 0.001)}{0.002}.$$

Some calculators have a name for the numerical derivative, like nDeriv$(f(x), x, a)$, which is similar to the one we use in the definition. Others use the Leibniz notation for the actual derivative at a:

$$\left.\frac{d}{dx}(f(x))\right|_{x=a}.$$

We do not want to suggest that the numerical derivative and the derivative are the same, so we will continue to use our generic term NDER when referring to the numerical derivative. Thus, in this textbook,

$$\text{NDER}\,(f(x), a) = \frac{f(a + 0.001) - f(a - 0.001)}{0.002},$$

while

$$\left.\frac{d}{dx}(f(x))\right|_{x=a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Be aware, however, that $\left.\frac{d}{dx}(f(x))\right|_{x=a}$ on your calculator might well refer to the numerical derivative.

EXAMPLE 2 Computing a Numerical Derivative

If $f(x) = x^3$, use the numerical derivative to approximate $f'(2)$.

SOLUTION

$$f'(2) = \left.\frac{d}{dx}(x^3)\right|_{x=2} \approx \text{NDER}(x^3, 2) = \frac{(2.001)^3 - (1.999)^3}{0.002} = 12.000001.$$

Now Try Exercise 17.

In Example 1 of Section 3.1, we found the derivative of x^3 to be $3x^2$, whose value at $x = 2$ is $3(2)^2 = 12$. The numerical derivative is accurate to 5 decimal places. Not bad for the push of a button.

Example 2 gives dramatic evidence that NDER is very accurate when $h = 0.001$. Such accuracy is usually the case, although it is also possible for NDER to produce some surprisingly inaccurate results, as in Example 3.

EXAMPLE 3 Fooling the Symmetric Difference Quotient

Compute NDER $(|x|, 0)$, the numerical derivative of $|x|$ at $x = 0$.

SOLUTION

We saw at the start of this section that $y = |x|$ is not differentiable at $x = 0$, since its right-hand and left-hand derivatives at $x = 0$ are not the same. Nonetheless,

$$\begin{aligned} \text{NDER}(|x|, 0) &= \frac{|0 + 0.001| - |0 - 0.001|}{2(0.001)} \\ &= \frac{0.001 - 0.001}{0.002} \\ &= 0 \end{aligned}$$

Even in the limit,

$$\lim_{h \to 0} \frac{|0 + h| - |0 - h|}{2h} = \lim_{h \to 0} \frac{0}{2h} = 0.$$

This proves that the derivative *cannot* be defined as the limit of the symmetric difference quotient. The symmetric difference quotient, which works on opposite sides of 0, has no chance of detecting the corner! *Now Try Exercise 23.*

In light of Example 3, it is worth repeating here that the symmetric difference quotient actually does approach $f'(a)$ *when* $f'(a)$ *exists,* and in fact approximates it quite well (as in Example 2).

EXPLORATION 2 Looking at the Symmetric Difference Quotient Analytically

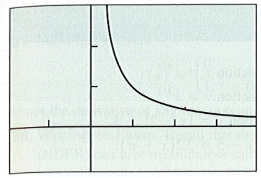
Let $f(x) = x^2$ and let $h = 0.01$.

1. Find
$$\frac{f(10 + h) - f(10)}{h}.$$
How close is it to $f'(10)$?
2. Find
$$\frac{f(10 + h) - f(10 - h)}{2h}.$$
How close is it to $f'(10)$?
3. Repeat this comparison for $f(x) = x^3$.

EXAMPLE 4 Graphing a Derivative Using NDER

Let $f(x) = \ln x$. Use NDER to graph $y = f'(x)$. Can you guess what function $f'(x)$ is by analyzing its graph?

continued



[–2, 4] by [–1, 3]
(a)

X	Y_1
.1	10
.2	5
.3	3.3333
.4	2.5
.5	2
.6	1.6667
.7	1.4286

X = .1

(b)

Figure 3.17 (a) The graph of NDER (ln (x), x) and (b) a table of values. What graph could this be? (Example 4)

SOLUTION

The graph is shown in Figure 3.17a. The shape of the graph suggests, and the table of values in Figure 3.17b supports, the conjecture that this is the graph of $y = 1/x$. We will prove in Section 3.9 (using analytic methods) that this is indeed the case. *Now Try Exercise 27.*

Differentiability Implies Continuity

We began this section with a look at the typical ways that a function could fail to have a derivative at a point. As one example, we indicated graphically that a discontinuity in the graph of f would cause one or both of the one-sided derivatives to be nonexistent. It is actually not difficult to give an analytic proof that continuity is an essential condition for the derivative to exist, so we include that as a theorem here.

THEOREM 1 Differentiability Implies Continuity

If f has a derivative at $x = a$, then f is continuous at $x = a$.

Proof Our task is to show that $\lim_{x \to a} f(x) = f(a)$, or, equivalently, that
$$\lim_{x \to a} [f(x) - f(a)] = 0.$$
Using the Limit Product Rule (and noting that $x - a$ is not zero), we can write
$$\begin{aligned} \lim_{x \to a} [f(x) - f(a)] &= \lim_{x \to a} \left[(x - a) \frac{f(x) - f(a)}{x - a} \right] \\ &= \lim_{x \to a} (x - a) \cdot \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \\ &= 0 \cdot f'(a) \\ &= 0. \end{aligned}$$
■

The converse of Theorem 1 is false, as we have already seen. A continuous function might have a corner, a cusp, or a vertical tangent line, and hence not be differentiable at a given point.

Intermediate Value Theorem for Derivatives

Not every function can be a derivative. A derivative must have the Intermediate Value Property, as stated in the following theorem (the proof of which can be found in advanced texts).

THEOREM 2 Intermediate Value Theorem for Derivatives

If a and b are any two points in an interval on which f is differentiable, then f' takes on every value between $f'(a)$ and $f'(b)$.

EXAMPLE 5 Applying Theorem 2

Does any function have the Unit Step Function (see Figure 3.14) as its derivative?

SOLUTION

No. Choose some $a < 0$ and some $b > 0$. Then $U(a) = -1$ and $U(b) = 1$, but U does not take on any value between -1 and 1. *Now Try Exercise 37.*

The question of when a function is a derivative of some function is one of the central questions in all of calculus. The answer, found by Newton and Leibniz, would revolutionize the world of mathematics. We will see what that answer is when we reach Chapter 6.

Quick Review 3.2 (For help, go to Sections 1.2 and 2.1.)

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator.*

In Exercises 1–5, tell whether the limit could be used to define $f'(a)$ (assuming that f is differentiable at a).

1. $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ **2.** $\lim_{h\to 0}\frac{f(a+h)-f(h)}{h}$

3. $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ **4.** $\lim_{x\to a}\frac{f(a)-f(x)}{a-x}$

5. $\lim_{h\to 0}\frac{f(a+h)+f(a-h)}{h}$

6. Find the domain of the function $y = x^{4/3}$.

7. Find the domain of the function $y = x^{3/4}$.

8. Find the range of the function $y = |x-2| + 3$.

9. Find the slope of the line $y - 5 = 3.2(x + \pi)$.

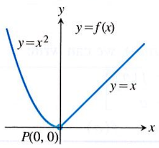
10. If $f(x) = 5x$, find

$$\frac{f(3+0.001)-f(3-0.001)}{0.002}.$$

Section 3.2 Exercises

In Exercises 1–4, compare the right-hand and left-hand derivatives to show that the function is not differentiable at the point P. Find all points where f is not differentiable.

1.



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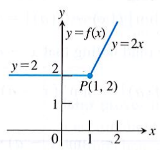

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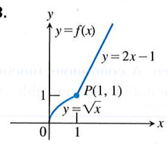

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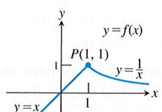
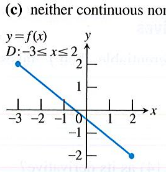


In Exercises 5–10, the graph of a function over a closed interval D is given. At what domain points does the function appear to be

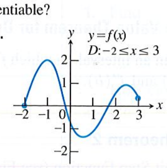
(a) differentiable? (b) continuous but not differentiable?

(c) neither continuous nor differentiable?

5.



6.



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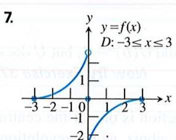

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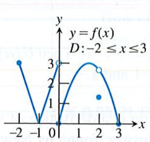
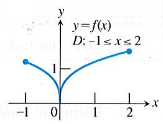
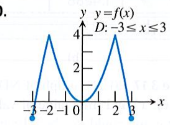


9.



10.



In Exercises 11–16, the function fails to be differentiable at $x = 0$. Tell whether the problem is a corner, a cusp, a vertical tangent, or a discontinuity.

11. $y = \begin{cases} \tan^{-1}x, & x \neq 0 \\ 1, & x = 0 \end{cases}$ **12.** $y = x^{4/5}$

13. $y = x + \sqrt{x^2} + 2$ **14.** $y = 3 - \sqrt[3]{x}$

15. $y = 3x - 2|x| - 1$ **16.** $y = \sqrt[3]{|x|}$

In Exercises 17–26, find the numerical derivative of the given function at the indicated point. Use $h = 0.001$. Is the function differentiable at the indicated point?

17. $f(x) = 4x - x^2, x = 0$ **18.** $f(x) = 4x - x^2, x = 3$

19. $f(x) = 4x - x^2, x = 1$ **20.** $f(x) = x^3 - 4x, x = 0$

21. $f(x) = x^3 - 4x, x = -2$ **22.** $f(x) = x^3 - 4x, x = 2$

23. $f(x) = x^{2/3}, x = 0$ **24.** $f(x) = |x - 3|, x = 3$

25. $f(x) = x^{2/5}, x = 0$ **26.** $f(x) = x^{4/5}, x = 0$

Group Activity In Exercises 27–30, use NDER to graph the derivative of the function. If possible, identify the derivative function by looking at the graph.

27. $y = -\cos x$ **28.** $y = 0.25x^4$

29. $y = \frac{x|x|}{2}$ **30.** $y = -\ln|\cos x|$

In Exercises 31–36, find all values of x for which the function is differentiable.

31. $f(x) = \frac{x^3-8}{x^2-4x-5}$ **32.** $h(x) = \sqrt[3]{3x-6} + 5$

33. $P(x) = \sin(|x|) - 1$ **34.** $Q(x) = 3\cos(|x|)$

35. $g(x) = \begin{cases} (x+1)^2, & x \le 0 \\ 2x+1, & 0 < x < 3 \\ (4-x)^2, & x \ge 3 \end{cases}$

36. $C(x) = x|x|$

37. Show that the function

$$f(x) = \begin{cases} 0, & -1 \le x < 0 \\ 1, & 0 \le x \le 1 \end{cases}$$

is not the derivative of any function on the interval $-1 \le x \le 1$.

38. Writing to Learn Recall that the numerical derivative (NDER) can give meaningless values at points where a function is not differentiable. In this exercise, we consider the numerical derivatives of the functions $1/x$ and $1/x^2$ at $x = 0$.

(a) Explain why neither function is differentiable at $x = 0$.

(b) Find NDER at $x = 0$ for each function.

(c) By analyzing the definition of the symmetric difference quotient, explain why NDER returns wrong responses that are so different from each other for these two functions.

39. Let f be the function defined as

$$f(x) = \begin{cases} 3 - x, & x < 1 \\ ax^2 + bx, & x \ge 1 \end{cases}$$

where a and b are constants.

(a) If the function is continuous for all x, what is the relationship between a and b?

(b) Find the unique values for a and b that will make f both continuous and differentiable.

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

40. True or False If f has a derivative at $x = a$, then f is continuous at $x = a$. Justify your answer.

41. True or False If f is continuous at $x = a$, then f has a derivative at $x = a$. Justify your answer.

42. Multiple Choice Which of the following is true about the graph of $f(x) = x^{4/5}$ at $x = 0$?

(A) It has a corner.

(B) It has a cusp.

(C) It has a vertical tangent.

(D) It has a discontinuity.

(E) $f(0)$ does not exist.

43. Multiple Choice Let $f(x) = \sqrt[3]{x} - 1$. At which of the following points is $f'(a) \neq \text{NDER}(f(x), x, a)$?

(A) $a = 1$ (B) $a = -1$ (C) $a = 2$ (D) $a = -2$

(E) $a = 0$

In Exercises 44 and 45, let

$$f(x) = \begin{cases} 2x + 1, & x \le 0 \\ x^2 + 1, & x > 0. \end{cases}$$

44. Multiple Choice Which of the following is equal to the left-hand derivative of f at $x = 0$?

(A) $2x$ (B) 2 (C) 0 (D) $-\infty$ (E) ∞

45. Multiple Choice Which of the following is equal to the right-hand derivative of f at $x = 0$?

(A) $2x$ (B) 2 (C) 0 (D) $-\infty$ (E) ∞

Explorations

46. (a) Enter the expression "$x < 0$" into Y1 of your calculator using "<" from the TEST menu. Graph Y1 in DOT MODE in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$.

(b) Describe the graph in part (a).

(c) Enter the expression "$x \ge 0$" into Y1 of your calculator using "≥" from the TEST menu. Graph Y1 in DOT MODE in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$.

(d) Describe the graph in part (c).

47. *Graphing Piecewise Functions on a Calculator* Let

$$f(x) = \begin{cases} x^2, & x \le 0 \\ 2x, & x > 0. \end{cases}$$

(a) Enter the expression "(X²)(X≤0)+(2X)(X>0)" into Y1 of your calculator and draw its graph in the window $[-4.7, 4.7]$ by $[-3, 5]$.

(b) Explain why the values of Y1 and $f(x)$ are the same.

(c) Enter the numerical derivative of Y1 into Y2 of your calculator and draw its graph in the same window. Turn off the graph of Y1.

(d) Use TRACE to calculate NDER(Y1, x, -0.1), NDER(Y1, x, 0), and NDER(Y1, x, 0.1). Compare with Section 3.1, Example 6.

Extending the Ideas

48. *Oscillation* There is another way that a function might fail to be differentiable, and that is by *oscillation.* Let

$$f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

(a) Show that f is continuous at $x = 0$.

(b) Show that

$$\frac{f(0+h)-f(0)}{h} = \sin\frac{1}{h}.$$

(c) Explain why

$$\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

does not exist.

(d) Does f have either a left-hand or right-hand derivative at $x = 0$?

(e) Now consider the function

$$g(x) = \begin{cases} x^2\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Use the definition of the derivative to show that g is differentiable at $x = 0$ and that $g'(0) = 0$.